interactions in detail before claiming one's research is of value. In effect, finally I will suggest another plausible argument, that one kind of method in the cognitive sciences is profoundly consonant with the methods of contemporary physics–and furthermore, our epistemological focus and methods could be improved by comparing problem solving in the two fields. My primary source for the physics of this comparison is a series of popular lectures by Richard Feynman, published in *QED: The Strange Theory of Light and Matter* (1985). Feynman was a primary architect of Quantum Electrodynamics (QED) and was its advocate as the most thorough and profound of current physical theories. The work in which this analysis is set forth is an outstanding model for enhancing the accessibility of science. It should occupy a central place in the library of anyone for whom the core issues of this paper are interesting.

Feynman's Focus: the Problem of Reflection

In a public lecture a week after Feynman's death, Marvin Minsky, a colleague and friend of Feynman's for many years, characterized one of his primary contributions to physics this way:

> "Richard Feynman's great originality was in reducing a substantial part of physics to a beautiful theory, called quantum electrodynamics, by deriving almost everything in that field from a single principle ..."
>
> M. Minsky, lecture at Purdue University, Feb. 28th 1988.

The book *QED* is based on a series of lectures given at UCLA.[8] The examples of Feynman's analysis–fascinating in their own right–can also help bring into

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[8] The editor of the transcripts, Ralph Leighton, remarks that:

"... [the] book is a venture that, as far as we know, has never been tried. It is a straightforward, honest explanation of a rather difficult subject–quantum electrodynamics–for a nontechnical audience. It is designed to give the interested focus the relationship between particular cases, the orderliness of mass phenomena, and possibilities of prediction. Here I want merely to present parts of Feynman's analysis. Feynman approached his theme through a discussion of the reflection of light from a surface. Reflection is a familiar phenomenon generally believed to be well understood by educated laymen, both in terms of common sense and at the academic level of classical physical theory. Feynman's analysis reveals that the classical theory does not account adequately for the phenomena as we know them today. The analysis even reveals what one might call "mysteries" unsolvable except through quantum analysis. Furthermore, the interpretation in terms of quantum electrodynamics permits directly the progressive deepening of layers of analysis to the level of atomic particle interactions. Additionally, it might even be the case that quantum electrodynamics is the best physical theory that exists, as Feynman himself claims in *QED* . [9]

There is a dilemma in classical electrodynamics in that one must, on the one hand, consider light as particles (called "photons"). Yet on the other hand, experimental results, refined over hundreds for years, have only been explainable in terms of the interference patterns of interacting waves. The sorts of experiments calling for a wave-theory based interpretation are exemplified in Figure 1. The evidence for light's particulate character is this: when very weak monochromotic light hits a dectector, the detector makes equally loud clicks less and less often as the light gets dimmer. In the first case of the experimental set up, a light source shines directly on a detector through a single surface of glass. 96% of the photons get to detector B and 4% are reflected to the detector A. In the second case, where the light goes through the two surfaces of a pane of glass, the result is strange. Instead of reflecting 4% of photons from the front and an additional 4% from the back

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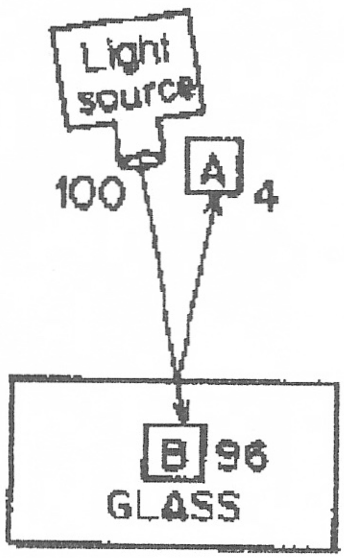
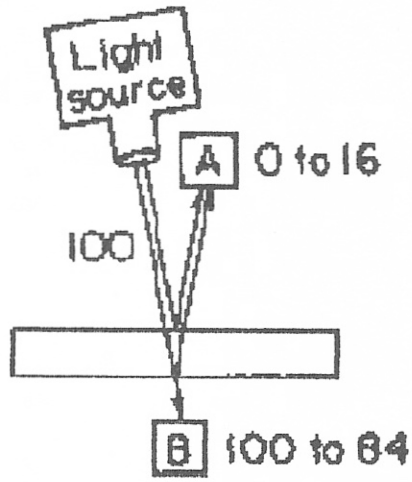
reader an appreciation for the kind of thinking that physicists have resorted to in order to explain how Nature behaves ..." *QED*, Preface, ix

[9] For his general characterization of the theory and the effort, See QED, p77 and pp37-38. (See Feyman.1)

surface, the amount of light relfected varies with the thickness of the glass. At a minimum of reflection, 100% of the photons get to detector B (and 0% get to A). At a maximum reflection, 84% get to B and 16% are reflected by the two glass surfaces.



One Reflecting Surface



Two Reflecting Surfaces

Figure1 [10]

Even though the calculations of wave theory -- based on waves interacting and "cancelling out" -- matched experimental results for hundreds of years after Newton first made such experiments, the theory broke down in modern times when detectors were developed sensitive enough to detect a single photon. The wave theory predicted that the "clicks" of the detector would get softer as the light got dimmer, whereas they actually stayed at full strength and just occurred less frequently. No reasonable model could explain this fact, so as Feynman puts it "...There was a period for a while where you had to be very clever: You had to know which experiment you were analyzing in order to tell if light was waves or particles. This state of confusion was called

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[10] Feynman's Figure 2, p.17, QED and Figure 4, p. 20, QED.

'the wave-particle duality of light.'... It is the purpose of these lectures to tell you how this puzzle was finally 'resolved'." Feynman goes on to relate how quantum electrodynamics explains the phenomena without claiming to actually "resolve" the dilemma down to an intuitive model based on experience in the everyday world. His description of this "resolution" [11] is important in characterizing his effort:

> " ... The situation today is, we haven't got a good model to explain partial reflection by two surfaces; we just calculate the probability that a particular photomultiplier will be hit by a photon reflected from a sheet of glass ... I am going to show you "how we count the beans"—what the physicists do to get the right answer. I am not going to explain how the photons actually "decide" whether to bounce back or go through; that is not known. (Probably the question has no meaning.) I will only show you how to calculate the correct probability that light will be reflected from glass of a given thickness, because that's the only thing physicists know how to do! What we do to get the answer to this problem is analogous to the things we have to do to get the answer to every other problem explained by quantum electrodynamics..." [12]

QED, Introduction, p. 24

Feynman: Considering All Ways - the Analysis

The probability that a particular photon will reflect off a glass surface is represented as a "probability amplitude" vector, which can be called an

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[11] I put quotes around the word "Resolution" here, as Feynman did above, because the problem is not resolved, in any sense of asserting that light is "really particles" or "really waves". We have no acceptable representation of the quantum particle situation based on the kinds of objects and behaviors we encounter in our everyday interactions with the world. The physical scale of photon interactions is so different from that at which our senses evolved there should be no surprise that objects and phenomena appear different in kind. Feynman makes a similar extended observation in the same vein in *The Character of Physical Law*, pp. 127-129. He takes the bull by the horns and argues that it is improper for us to expect any such "resolution" of the problem. In a chapter of *The Character of Physical Law* describing the relation of Mathematics to Physics, Feynman makes a similar point about Newton's refusal to say what the Law of Gravitation means (p. 37).

[12] For some of the details of Feynman's representation used in the discussion, see QED p. 23 and p. 36. (See Feynman.4)

arrow. An arrow represents, for each photon, the probability that it will reflect from the glass surface. The direction of the arrow is a function of the time elapsed since the photon left the emitter. It speeds around like a very quick clock hand. Feynman uses that representation in the analysis of reflection from a mirror such as that show in the top part of Figure 2. [13]

> "Although you might think that the parts of the mirror near the two ends have nothing to do with the reflection of the light that goes from the source to the detector, (Figure 2.a) let us look at what quantum theory has to say. **Rule I**: the probability that a particular event occurs is the square of a final arrow that is found by drawing an arrow for each way the event could happen and then combining ("adding") the arrows. In the experiment measuring the partial reflection of light by two surfaces (Figure 1), there were two ways a photon could get from the source to the detector. In this experiment, there are millions of ways a photon could go: it could go down to the left-hand part of the mirror at A or B (for example) and bounce up to the detector ...; it could bounce off the part where you think it should, at G or, it could go down to the right-hand part of the mirror at K or M and bounce up to the detector. You might think I'm crazy, because for most of the ways I told you a photon could reflect off the mirror, the angles aren't equal. But I'm not crazy, because that's the way light really goes! How can that be?..."
>
> "Although it is safe to assume that the length to all the arrows will be nearly the same,[14] their directions will clearly differ because their timing is different ... the direction of a particular arrow is determined by the final position of an imaginary stop-watch that times a photon as it moves along that particular path. When a photon goes way off to the left end of the mirror, at A, and then up to the detector, it clearly takes more time than a photon that gets to the detector by reflecting in the middle of the mirror, at G..."

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[13] The following extensive quotations are all drawn from Chapter 2, *Photons: Particles of Light* , beginning at page 38.

[14] All will have a length of .02 so that the square for each will be .04 or 4% for the mass.

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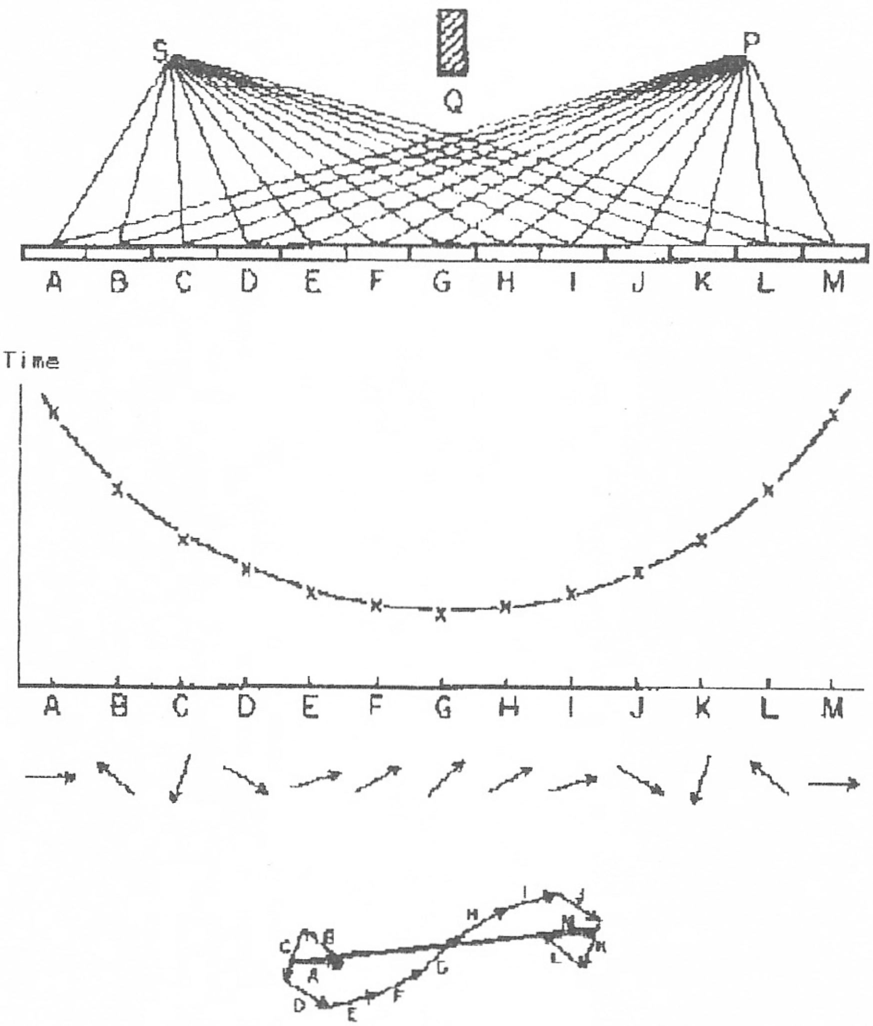

Figure 2 [15]

The three components of Figure 2 are referred to in the accompanying quotations as figures 2a, 2b, & 2c. [16]

> "... To help us calculate the direction of each arrow, I'm going to draw a graph right underneath my sketch of the mirror (see Fig. 2.b). Directly below each place on the mirror where the light could reflect, I'm going to show, vertically, how much time it would take if the light went that way. The more time it takes, the higher the point will be on the graph. Starting at the left,

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[15] Feynman's Figure 24 from p. 43 QED.

[16] The text of these quotations begin on page 42 of QED and continues through page 45.

the time it takes a photon to go on the path that reflects at A is pretty long, so we plot a point pretty high up on the graph. As we move toward the center of the mirror, the time it takes for a photon to go the particular way we're looking at goes down, so we plot each successive point lower than the previous one. After we pass the center of the mirror, the time it takes a photon to go on each successive path gets longer and longer, so we plot our points correspondingly higher and higher. To aid the eye, let's connect the points: they form a symmetrical curve that starts high, goes down, and then goes back up again..."

"...Now, what does that mean for the direction of the little arrows? The direction of a particular arrow corresponds to the amount of time it would take a photon to get from the source to the detector following that particular path. Let's draw the arrows, starting at the left. Path A takes the most time; its arrow points in some direction (Fig. 2.c). The arrow for path B points in a different direction because its time is different. At the middle of the mirror, arrows F, G, and H point in nearly the same direction because their times are nearly the same. After passing the center of the mirror, we see that each path on the right side of the mirror corresponds to a path on the left side whose time is exactly the same (this is a consequence of putting the source and the detector at the same height, and path G exactly in the middle). Thus the arrow for path J, for example, has the same direction as the arrow for path D.

"Now, let's add the little arrows (Fig. 2.c). Starting with arrow A, we hook the arrows to each other, head to tail. Now, if we were to take a walk using each little arrow as a step, we wouldn't get very far at the beginning, because the direction from one step to the next is so different. But after a while the arrows begin to point in generally the same direction, and we make some progress. Finally, near the end of our walk, the direction from one step to the next is again quite different, so we stagger about some more.

"All we have to do now is draw the final arrow. We simply connect the tail of the first little arrow to the head of the last one, and see how much direct progress we made on our walk (Fig. 2.c). And behold—we get a sizable final arrow! The theory of quantum electrodynamics predicts that light does, indeed, reflect off the mirror!

"Now, let's investigate. What determines how long the final arrow is? We notice a number of things. First, the ends of the mirror are not important: there, the little arrows wander around and don't get anywhere. If I chopped off the ends of the mirror—parts that you instinctively knew I was wasting my time fiddling around with—it would hardly affect the length of the final arrow. So where is the part of the mirror that gives the final arrow a substantial length? It's the part where the arrows are all pointing in nearly the same direction—because their time is almost the same. If you look at the graph showing the time for each path (Fig. 2.b), you see that the time is

> nearly the same from one path to the next at the bottom of the curve, where the time is least.
>
> "To summarize, where the time is least is also where the time for the nearby paths is nearly the same; that's where the little arrows point in nearly the same direction and add up to a substantial length; that's where the probability of a photon reflecting off a mirror is determined. And that's why, in approximation, we can get away with the crude picture of the world that says that light only goes where the time is least...
>
> "So the theory of quantum electrodynamics gave the right answer—the middle of the mirror is the important part for reflection—but this correct result came out at the expense of believing that light reflects all over the mirror, and having to add a bunch of little arrows together whose sole purpose was to cancel out. All that might seem to you to be a waste of time—some silly game for mathematicians only. After all, it doesn't seem like "real physics" to have something there that only cancels out! ..."

Comments on the First Example from Feynman's Analysis

Feynman's analysis highlights some specific relationships between the study of particular cases and the formulation of general principles. Although there are limitations inherent in studies of particular cases, such studies are central to the detailed analysis necessary for illuminating the explicit meaning of general principles. One may even argue that the cases frequently come first, as particular problems to be solved, and that from their solutions general principles emerge, which in turn are finally comprehensible when applied through subsequent models. This argument is consonant with the views of Weyl in *Symmetry* and Bourbaki in *The Architecture of Mathematics*.[17]

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[17] One might imagine that in mathematics, considered the most abstract of disciplines, this judgment is wrong. Such is not obvious at all. Bourbaki's description of the genesis of an axiomatic system shows a specific role for particular analysis in development of very general systems:

"A mathematician who tries to carry out a proof thinks of a well-defined mathematical object, which he is studying just at this moment. If he now believes that he has found a proof, he notices then, as he carefully examines all the sequences of inference, that only very few of the special properties in the object at issue have really played any significant role in the proof. It is consequently possible to carry out the same proof also for other objects possessing only those properties which had to be used. Here lies the simple idea of the axiomatic method: instead of explaining which objects should be

Particular cases provide us guidance in both the formulation and comprehension of general laws.

Lawler's Focus: Learning through Interaction

It is remarkable in Feynman's discussion that something so familiar as reflection from a mirror is a pathway into solving the deepest puzzles of physics, as in wave-particle duality.[18] And yet, why be surprised ? Insights are usually a reconceptualization of familiar affairs. Learning is something we have all experienced personally for long periods of time, something we see in others all the time. There are epistemological and psychological reasons to believe that a case-based approach is better suited than lab-based methods for gathering information about developmental issues, such as the character of learning.[19] Specifically, if one sees learning as an adaptive developmental mechanism, then one should look at learning where it happens in the everyday world. Furthermore, if learning is a process of changing one state of a cognitive system to another, then representations of that process in computing terms should be expected to be more apt than in other schemes

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examined, one has to specify only the properties of the objects which are to be used. These properties are placed as axioms at the start. It is no longer necessary to explain what the objects that should be studied really are."

N. Bourbaki, in J. Fang, p. 69.

[18] See the later discussion under the heading "Feynman: on the Reality of Reflection All Ways."

[19] If one recalls that case study is the method underlying the theories of Freud and Piaget and that ecological studies such as those of Barker and Wright (1951) and, in our our decades, the stunning work of Goodall (1971, 1990), it is not hard to believe that the fusion of such methods may continue to help us learn about human development. Ecologically oriented studies, such as those of Barker and Wright (1951, 1967), pay close attention to the context of behavior. That context of behavior is also the primary situation in which learning takes place and thus should be considered in detail in any study of learning through interaction.