permitted in the simulations. These descent networks are summaries of results.

Comparison and Contrast

The apparent similarities relating Feynman's analysis of reflection and the exploratory epistemology of SLIM occur at different levels. They begin with a focus on detail:

- in the analysis of specific cases and
- in the analysis of the interaction of objects or agents with their context.

The core principle applied in both is to try all cases and construct an interpretation of them. There are many paths of possible learning, some central and some peripheral. In QED, the criterion of centrality is near-uniform directionality of the photon arrows. In SLIM, the criterion of centrality is a different and a new one: **co-generativity**. The core method is to aggregate results of all possibilities in a fully explicit manner. The process of aggregation is where the differences become systematic and significant. In QED, the aggregation of individual results is formally analytic–that is the solution of path-integral equations of functions of complex variables [31]. In

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[31] The Preface to *Quantum Mechanics and Path Integrals* (1965) provides a little history which puts the descriptions used here in their proper perspective:

"The fundamental physical and mathematical concepts which underlie the path integral approach to quantum mechanics were first developed by R. P. Feynman in the course of his graduate studies at Princeton.... These early inquiries were involved with the problem of the infinite self-energy of the electron. In working on that problem, a "least-action" principal using half-advanced and half-retarded potentials was discovered. The principle could deal successfully with the infinity arising in the application of classical electrodynamics.

The problem then became one of applying this action principle to quantum mechanics in such a way that classical mechanics could arise naturally as a special case of quantum mechanics when h was allowed to go to zero.

SLIM, one begins with game lists and then reformulate the relations in a tree-like representation. Fork-plan transformations comprise the trees of descent; the learning algorithms are the functional mechanisms effecting the transformations.[32] In the analysis of SLIM, further aggregation is systematic and constructive if not formal: one pulls together the empirical results of exhaustive exploration (trees of descent in figure 4) into a new representation scheme (the genetic networks of descent in figures 5 and 6).

In general, the process followed with these data is similar to Weyl's use of reformulation in his general description of the development of theoretic knowledge in *Symmetry* and Bourbaki's description of the genesis of axiomatic systems in *The Architecture of Mathematics*. Working with SLIM started with the general principle that learning happens through interaction. The model was constructed to represent the behavior of both the learner and the opponent in explicit detail with specification of representations and learning algorithms giving the notion a precise meaning. Through the aggregation and reformulation of results, a new generality was achieved which suggested a new idea -- that co-generability of related but variant knowledge forms is what makes learning possible in any particular domain.

Feynman searched for any ideas which might have been previously worked out in connecting quantum-mechanical behavior with such classical ideas as the lagrangian or, in particular, Hamilton's principle function S, the indefinite integral of the lagrangian. During some conversations with a visiting European physicist, Feynman learned of a paper in which Dirac had suggested that the exponential function of *i e* [epsilon] times the lagrangian was analagous to a transformation function for the quantum-mechanical wave function in that the wave function at one moment could be related to the wave function at the next moment (a time interval e [episilon] later) by multiplying with such an exponential function.

The question that then arose was what Dirac had meant by the phrase "analagous to," and Feynman determined to find out whether or not it would be possible to substitute the phrase "equal to." A brief analysis showed that indeed this exponential function could be used in this manner directly."

[32] The conclusion of the analysis is independent of the implementation, and so it would be limiting its scope if one claimed that it represented human learning only -- though that may remain the most important case for most of us.

Furthermore, of course, QED claims completeness in covering all known interactions of photons and electrons. Strategy learning is important, possibly even central in studies of cognition; but it would be hard to support an argument that it is as fundamental in cognition as electron-photon interactions are in physics. Moreover, quantum electrodynamics permits going further down in detail to the discussion of interactions of such particles in the atomic matrix of matter. The comparable deeper level for epistemology would be biopsychology or neurophysiology, which I do not pretend to approach here.

In respect of ways for aggregating results, the similarities are superficial unless the consequences are similar or significant for some other reason. What are the results of Feynman's analysis? Two results at least are important in their use for us. Feynman says that one must regularly remind students of the goal of the process–discovering "the final arrow," that is, the resultant vector of probability amplitudes, and understanding how the particularities of the interaction generate that "final arrow." Our objective is comparable. The learnability analysis of SLIM introduces two novelties: a new goal and a new principle. The **new goal** transforms a psychological focus to an epistemological one. At this point one is not so much interested in what a particular child did as an individual. But the case provides some boundary conditions for modelling with the question "if at least one natural case has such characteristics, what kinds and paths of learning are possible?" The question is of general interest if one admits that particularity and egocentricity are common characteristics of novice thought.

Where Feynman's analysis of reflection depends on the principle of least time to determine what paths contribute most to the observable "final arrow," SLIM points to cogenerativity, represented by those connections in the central group (strategies 1-6) permitting each one to be learned no matter which is adopted as the prototype fork. The peripheral strategies (13 and 14 of Figure 6 for example) are rarely learned because they can be learned in only one way. The conclusion is that one can characterize the learnability of a domain as a function of particular interactions among agents based on the connectedness of possible paths of strategy learning. This is what the network of genetic descent does. That network is the equivalent of the "final arrow" of Feynman's analysis of reflection, despite its different appearance and different mathematical basis.

Furthermore, these methods and representations reveal what makes it possible to learn in a problem domain; even what makes it possible to judge that knowledge of a given domain is more learnable in one context than in another. The **new principle** is that the learnability of a domain is the result of all the possible cases of concrete learning through particular experience; those that contribute significantly to the learnability of a domain are those that are mutually reinforcing, in the sense that they are co-generative: A -> B -> C -> A, etc.[33] This is directly comparable to what one finds the case in addition of arrows -- the final effect results from the aggregation of related components; isolated possibilities don't add up to a significant result. The new principle -- identifying the learnability of a domain with its connectedness -- is a direct consequence of the state transformations being the learning algorithms of the system.

At the end of his discussion of the problem of reflection, Feynman jokes that it doesn't "seem like real physics" to add bunches of little arrows (representing the off-center parts of the mirror's reflection) because they will only cancel out in the end. What then is the "real physics"? Is it a compact and simple description that one can use and rely on? Or is it a thorough consideration of all the cases, even including the improbable ones that may in special circumstances yield decisive results? The former is the result one hopes for: confidence in a succinct description that is so broadly applicable we

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[33] It is important to distinguish between the learnability of a particular domain *per se*, and the subsequent ramification of knowledge based on learning in it.

might call it a principle or law. The latter is a process that can increase our confidence in what we know, even if the result is not so general as to be judged a "law". That's what science is about [34].

Feynman: on The Reality of Reflection All Ways

There are many cases where the claim to reality and direct fitness to the world is not essential, but physics is a field where the claim to reality <u>is</u> central. Feyman's explanation of how a diffraction grating can scatter a flood of particles in a way formerly only comprehensible through the wave theory of light shows the power of the QED formulation. It also supports the reality claim of his central principle -- that the orderliness of light phenomena emerges as a function of the individual probabilities of masses of photons going all possible ways.

To illuminate the cases where "interference" occurs, Feynman proposes an experiment -- to examine a case where reflection "shouldn't happen" -- far out on a mirror from the center line between a light source and photon counter. He selects the slices A and B from the left hand side of the mirror in figure 2 and turns to examining the case in a finer grain of detail.

> "... In this experiment I am going to make a more detailed calculation by taking intervals on that left-hand part of the mirror that are much closer together—fine enough that there is not much difference in time between neighboring paths (see Fig. 7). With this more detailed picture, we see that some of the arrows point more or less to the right; the others point more or less to the left. If we add all the arrows together, we have a bunch of arrows going around in what is essentially a circle, getting nowhere."

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[34] Although this statement may strike some readers as insufficiently rigorous, it reflects in short compass the view advanced by C.S.Peirce in *The Fixation of Belief* and *Lessons from the History of Science.*